# Cartan For Beginners Differential Geometry Via Moving Frames And Exterior Differential Systems Graduate Studies In Mathematics

This volume contains original papers and announcements of recent results presented by the main participants of the 5th International Colloquium on Differential Geometry and its Related Fields (ICDG2016). These articles are devoted to some new developments on geometric structures on manifolds. Besides covering a broad overview on geometric structures, this volume provides significant information for researchers not only in the field of differential geometry but also in mathematical physics. Since each article is accompanied with detailed explanations, it serves as a good guide for young scientists working in this area.

This is a book that the author wishes had been available to him when he was student. It reflects his interest in knowing (like expert mathematicians) the most relevant mathematics for theoretical physics, but in the style of physicists. This means that one is not facing the study of a collection of definitions, remarks, theorems, corollaries, lemmas, etc. but a narrative — almost like a story being told — that does not impede sophistication and deep results. It covers differential geometry far beyond what general relativists perceive they need to know. And it introduces readers to other areas of mathematics that are of interest to physicists and mathematicians, but are largely overlooked. Among these is Clifford Algebra and its uses in conjunction with differential forms and moving frames. It opens new research vistas that expand the subject matter. In an appendix on the classical theory of curves and surfaces, the author slashes not only the main proofs of the traditional approach, which uses vector calculus, but even existing treatments that also use differential forms for the same purpose.

Contents:Introduction:OrientationsTools:Differential FormsVector Spaces and Tensor ProductsExterior DifferentiationTwo Klein Geometries:Affine Klein GeometryEuclidean Klein GeometryCartan Connections:Generalized Geometry Made SimpleAffine ConnectionsEuclidean ConnectionsRiemannian Spaces and Pseudo-SpacesThe Future?:Extensions of CartanUnderstand the Past to Imagine the FutureA Book of Farewells Readership: Physicists and mathematicians working on differential geometry. Keywords:Differential Geometry;Differential Forms;Moving Frames;Exterior CalculusKey Features:Reader FriendlyNaturalnessRespect for the history of the subject and related incisiveness

This textbook is suitable for a one semester lecture course on differential geometry for students of mathematics or STEM disciplines with a working knowledge of analysis, linear algebra, complex analysis, and point set topology. The book treats the subject both from an extrinsic and an intrinsic view point. The first chapters give a historical overview of the field and contain an introduction to basic concepts such as manifolds and smooth maps, vector fields and flows, and Lie groups, leading up to the theorem of Frobenius. Subsequent chapters deal with the Levi-Civita connection, Geodesics, the Riemann curvature tensor, a proof of the Cartan-Ambrose-Hicks theorem, as well as applications to flat spaces, symmetric spaces, and constant curvature manifolds. Also included are sections about manifolds with nonpositive sectional curvature, the Ricci tensor, the scalar curvature, and the Weyl tensor. An additional chapter goes beyond the scope of a one semester lecture course and deals with subjects such as conjugate points and the Morse index, the injectivity radius, the group of isometries and the Myers-Steenrod theorem, and Donaldson's differential geometric approach to Lie algebra theory.

This book introduces readers to the living topics of Riemannian Geometry and details the main results known to date. The results are stated without detailed proofs but the main ideas involved are described, affording the reader a sweeping panoramic view of almost the entirety of the field. From the reviews "The book has intrinsic value for a student as well as for an experienced geometer. Additionally, it is really a compendium in Riemannian Geometry." --MATHEMATICAL REVIEWS

The method of moving frames originated in the early nineteenth century with the notion of the Frenet frame along a curve in Euclidean space. Later, Darboux expanded this idea to the study of surfaces. The method was brought to its full power in the early twentieth century by Elie Cartan, and its development continues today with the work of Fels, Olver, and others. This book is an introduction to the method of moving frames as developed by Cartan, at a level suitable for beginning graduate students familiar with the geometry of curves and surfaces in Euclidean space. The main focus is on the use of this method to compute local geometric invariants for curves and surfaces in various 3-dimensional homogeneous spaces, including Euclidean, Minkowski, equi-affine, and projective spaces. Later chapters include applications to several classical problems in differential geometry, as well as an introduction to the nonhomogeneous case via moving frames on Riemannian manifolds. The book is written in a reader-friendly style, building on already familiar concepts from curves and surfaces in Euclidean space. A special feature of this book is the inclusion of detailed guidance regarding the use of the computer algebra system Maple™ to perform many of the computations involved in the exercises.

This volume presents lectures given at the Wisa 19 Summer School: Differential Geometry, Differential Equations, and Mathematical Physics, which took place from August 19 - 29th, 2019 in Wis?a, Poland, and was organized by the Baltic Institute of Mathematics. The lectures were dedicated to symplectic and Poisson geometry, tractor calculus, and the integration of ordinary differential equations, and are included here as lecture notes comprising the first three chapters. Following this, chapters combine theoretical and applied perspectives to explore topics at the intersection of differential geometry, differential equations, and mathematical physics. Specific topics covered include: Parabolic geometry Geometric methods for solving PDEs in physics, mathematical biology, and mathematical finance Darcy and Euler flows of real gases Differential invariants for fluid and gas flow Differential Geometry, Differential Equations, and Mathematical Physics is ideal for graduate students and researchers working in these areas. A basic understanding of differential geometry is assumed.

Elementary, yet authoritative and scholarly, this book offers an excellent brief introduction to the classical theory of differential geometry. It is aimed at advanced undergraduate and graduate students who will find it not only highly readable but replete with illustrations carefully selected to help stimulate the student's visual understanding of geometry. The text features an abundance of problems, most of which are simple enough for class use, and often convey an interesting geometrical fact. A selection of more difficult problems has been included to challenge the ambitious student. Written by a noted mathematician and historian of mathematics, this volume presents the fundamental conceptions of the theory of curves and surfaces and applies them to a number of examples. Dr. Struik has enhanced the treatment with copious historical, biographical, and bibliographical references that place the theory in context and encourage the student to consult original sources and discover additional important ideas there. For this second edition, Professor Struik made some corrections and added an appendix with a sketch of the application of Cartan's method of Pfaffians to curve and surface theory. The result was to further increase the merit of this stimulating, thought-provoking text — ideal for classroom use, but also perfectly suited for self-study. In this attractive, inexpensive paperback edition, it belongs in the library of any mathematician or student of mathematics interested in differential geometry.

Diffeology is an extension of differential geometry. With a minimal set of axioms, diffeology allows us to deal simply but rigorously with objects which do not fall within the usual field of differential geometry: quotients of manifolds (even non-Hausdorff), spaces of functions, groups of diffeomorphisms, etc. The category of diffeology objects is stable under standard set-theoretic operations, such as quotients, products, co-products, subsets, limits, and co-limits. With its right balance between rigor and simplicity, diffeology can be a good framework for many problems that appear in various areas of physics. Actually, the book lays the foundations of the main fields of differential geometry used in theoretical physics: differentiability, Cartan differential calculus, homology and cohomology, diffeological groups, fiber bundles, and connections. The book ends with an open program on symplectic diffeology, a rich field of application of the theory. Many exercises with solutions make this book appropriate for learning the subject.

Cartan geometries were the first examples of connections on a principal bundle. They seem to be almost unknown these days, in spite of the great beauty and conceptual power they confer on geometry. The aim of the present book is to fill the gap in the literature on differential geometry by the missing notion of Cartan connections. Although the author had in mind a book accessible to graduate students, potential readers would also include working differential geometers who would like to know more about what Cartan did, which was to give a notion of "espaces généralisés" (= Cartan geometries) generalizing homogeneous spaces (= Klein geometries) in the same way that Riemannian geometry generalizes Euclidean geometry. In addition, physicists will be interested to see the fully satisfying way in which their gauge theory can be truly regarded as geometry.

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Parabolic geometries encompass a very diverse class of geometric structures, including such important examples as conformal, projective, and almost quaternionic structures, hypersurface type CR-structures and various types of generic distributions. The characteristic feature of parabolic geometries is an equivalent description by a Cartan geometry modeled on a generalized flag manifold (the quotient of a semisimple Lie group by a parabolic subgroup). Background on differential geometry, with a view towards Cartan connections, and on semisimple Lie algebras and their representations, which play a crucial role in the theory, is collected in two introductory chapters. The main part discusses the equivalence between Cartan connections and underlying structures, including a complete proof of Kostant's version of the Bott - Borel - Weil theorem, which is used as an important tool. For many examples, the complete description of the geometry and its basic invariants is worked out in detail. The constructions of correspondence spaces and twistor spaces and analogs of the Fefferman construction are presented both in general and in several examples. The last chapter studies Weyl structures, which provide classes of distinguished connections as well as an equivalent description of the Cartan connection in terms of data associated to the underlying geometry. Several applications are discussed throughout the text.

This three-week summer program considered the symmetries preserving various natural geometric structures. There are two parts to the proceedings. The articles in the first part are expository but all contain significant new material. The articles in the second part are concerned with original research. All articles were thoroughly refereed and the range of interrelated work ensures that this will be an extremely useful collection.

Expert treatment introduces semi-Riemannian geometry and its principal physical application, Einstein's theory of general relativity, using the Cartan exterior calculus as a principal tool. Prerequisites include linear algebra and advanced calculus. 2012 edition.

This book surveys the differential geometry of varieties with degenerate Gauss maps, using moving frames and exterior differential forms as well as tensor methods. The authors illustrate the structure of varieties with degenerate Gauss maps, determine the singular points and singular varieties, find focal images and construct a classification of the varieties with degenerate Gauss maps.

The book is devoted to the study of the geometrical and topological structure of gauge theories. It consists of the following three building blocks:- Geometry and topology of fibre bundles,- Clifford algebras, spin structures and Dirac operators,- Gauge theory.Written in the style of a mathematical textbook, it combines a comprehensive presentation of the mathematical foundations with a discussion of a variety of advanced topics in gauge theory.The first building block includes a number of specific topics, like invariant connections, universal connections, H-structures and the Postnikov approximation of classifying spaces.Given the great importance of Dirac operators in gauge theory, a complete proof of the Atiyah-Singer Index Theorem is presented. The gauge theory part contains the study of Yang-Mills equations (including the theory of instantons and the classical stability analysis), the discussion of various models with matter fields (including magnetic monopoles, the Seiberg-Witten model and dimensional reduction) and the investigation of the structure of the gauge orbit space. The final chapter is devoted to elements of quantum gauge theory including the discussion of the Gribov problem, anomalies and the implementation of the non-generic gauge orbit strata in the framework of Hamiltonian lattice gauge theory.The book is addressed both to physicists and mathematicians. It is intended to be accessible to students starting from a graduate level.

An inviting, intuitive, and visual exploration of differential geometry and forms Visual Differential Geometry and Forms fulfills two principal goals. In the first four acts, Tristan Needham puts the geometry back into differential geometry. Using 235 hand-drawn diagrams, Needham deploys Newton's geometrical methods to provide geometrical explanations of the classical results. In the fifth act, he offers the first undergraduate introduction to differential forms that treats advanced topics in an intuitive and geometrical manner. Unique features of the first four acts include: four distinct geometrical proofs of the fundamentally important Global Gauss-Bonnet theorem, providing a stunning link between local geometry and global topology; a simple, geometrical proof of Gauss's famous Theorema Egregium; a complete geometrical treatment of the Riemann curvature tensor of an n-manifold; and a detailed geometrical treatment of Einstein's field equation, describing gravity as curved spacetime (General Relativity), together with its implications for gravitational waves, black holes, and cosmology. The final act elucidates such topics as the unification of all the integral theorems of vector calculus; the elegant reformulation of Maxwell's equations of electromagnetism in terms of 2-forms; de Rham cohomology; differential geometry via Cartan's method of moving frames; and the calculation of the Riemann tensor using curvature 2-forms. Six of the seven chapters of Act V can be read completely independently from the rest of the book. Requiring only basic calculus and geometry, Visual Differential Geometry and Forms provocatively rethinks the way this important area of mathematics should be considered and taught.

Conformal invariants (conformally invariant tensors, conformally covariant differential operators, conformal holonomy groups etc.) are of central significance in differential geometry and physics. Well-known examples of such operators are the Yamabe-, the Paneitz-, the Dirac- and the twistor operator. The aim of the seminar was to present the basic ideas and some of the recent developments around Q-curvature and conformal holonomy. The part on Q-curvature discusses its origin, its relevance in geometry, spectral theory and physics. Here the influence of ideas which have their origin in the AdS/CFT-correspondence becomes visible. The part on conformal holonomy describes recent classification results, its relation to Einstein metrics and to conformal Killing spinors, and related special geometries.

This classic and long out of print text by the famous French mathematician Henri Cartan, has finally been retitled and reissued as an unabridged reprint of the Kershaw Publishing Company 1971 edition at remarkably low price for a new generation of university students and teachers. It provides a concise and beautifully written course on rigorous analysis. Unlike most similar texts, which usually develop the theory in either metric or Euclidean spaces, Cartan's text is set entirely in normed vector spaces, particularly Banach spaces. This not only allows the author to develop carefully the concepts of calculus in a setting of maximal generality, it allows him to unify both single and multivariable calculus over either the real or complex scalar fields by considering derivatives of nth orders as linear transformations. This prepares the student for the subsequent study of differentiable manifolds modeled on Banach spaces as well as graduate analysis courses, where normed spaces and their isomorphisms play a central role. More importantly, it's republication in an inexpensive edition finally makes available again the English translations of both long separated halves of Cartan's famous 1965-6 analysis course at the University of Paris: The second half has been in print for over a decade as Differential Forms , published by Dover Books. Without the first half, it has been very difficult for readers of that second half text to be prepared with the proper prerequisites as Cartan originally intended. With both texts now available at very affordable prices, the entire course can now be easily obtained and studied as it was originally intended. The book is divided into two chapters. The first develops the abstract differential calculus. After an introductory section providing the necessary background on the elements of Banach spaces, the Frechet derivative is defined, and proofs are given of the two basic theorems of differential calculus: The mean value theorem and the inverse function theorem. The chapter proceeds with the introduction and study of higher order derivatives and a proof of Taylor's formula. It closes with a study of local maxima and minima including both necessary and sufficient conditions for the existence of such minima. The second chapter is devoted to differential equations. Then the general existence and uniqueness theorems for ordinary differential equations on Banach spaces are proved. Applications of this material to linear equations and to obtaining various properties of solutions of differential equations are then given. Finally the relation between partial differential equations of the first order and ordinary differential equations is discussed. The prerequisites are rigorous first courses in calculus on the real line (elementary analysis), linear algebra on abstract vectors spaces with linear transformations and the basic definitions of topology (metric spaces, topology,etc.) A basic course in differential equations is advised as well. Together with its' sequel, Differential Calculus On Normed Spaces forms the basis for an outstanding advanced undergraduate/first year graduate analysis course in the Bourbakian French tradition of Jean Dieudonn's Foundations of Modern Analysis, but a more accessible level and much more affordable then that classic.

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This classic work is now available in an unabridged paperback edition. Stoker makes this fertile branch of mathematics accessible to the nonspecialist by the use of three different notations: vector algebra and calculus, tensor calculus, and the notation devised by Cartan, which employs invariant differential forms as elements in an algebra due to Grassman, combined with an operation called exterior differentiation. Assumed are a passing acquaintance with linear algebra and the basic elements of analysis.

Cartan for BeginnersAmerican Mathematical Soc.

This book gives a treatment of exterior differential systems. It will in clude both the general theory and various applications. An exterior differential system is a system of equations on a manifold defined by equating to zero a number of exterior differential forms. When all the forms are linear, it is called a pfaffian system. Our object is to study its integral manifolds, i. e. , submanifolds satisfying all the equations of the system. A fundamental fact is that every equation implies the one obtained by exterior differentiation, so that the complete set of equations associated to an exterior differential system constitutes a differential ideal in the algebra of all smooth forms. Thus the theory is coordinate-free and computations typically have an algebraic character; however, even when coordinates are used in intermediate steps, the use of exterior algebra helps to efficiently guide the computations, and as a consequence the treatment adapts well to geometrical and physical problems. A system of partial differential equations, with any number of inde pendent and dependent variables and involving partial derivatives of any order, can be written as an exterior differential system. In this case we are interested in integral manifolds on which certain coordinates remain independent. The corresponding notion in exterior differential systems is the independence condition: certain pfaffian forms remain linearly indepen dent. Partial differential equations and exterior differential systems with an independence condition are essentially the same object.

This book lets readers understand differential geometry with differential forms. It is unique in providing detailed treatments of topics not normally found elsewhere, like the programs of B. Riemann and F. Klein in the second half of the 19th century, and their being superseded by E. Cartan in the twentieth. Several conservation laws are presented in a unified way. The Einstein 3-form rather than the Einstein tensor is emphasized; their relationship is shown. Examples are chosen for their pedagogic value. Numerous advanced comments are sprinkled throughout the text. The equations of structure are addressed in different ways. First, in affine and Euclidean spaces, where torsion and curvature simply happen to be zero. In a second approach, the 2-torus and the punctured plane and 2-sphere are endowed with the "Columbus connection," torsion becoming a concept which could have been understood even by sailors of the 15th century. Those equations are then presented as the breaking of integrability conditions for connection equations. Finally, a topological definition brings together the concepts of connection and equations of structure. These options should meet the needs and learning objectives of readers with very different backgrounds. Dr Howard E Brandt

Topics in Differential Geometry is a collection of papers related to the work of Evan Tom Davies in differential geometry. Some papers discuss projective differential geometry, the neutrino energy-momentum tensor, and the divergence-free third order concomitants of the metric tensor in three dimensions. Other papers explain generalized Clebsch representations on manifolds, locally symmetric vector fields in a Riemannian space, mean curvature of immersed manifolds, and differential geometry of totally real submanifolds. One paper considers the symmetry of the first and second order for a vector field in a Riemannnian space to arrive at conditions the vector field satisfies. Another paper examines the concept of a smooth manifold-tensor and the three types of connections on the tangent bundle TM, their properties, and their inter-relationships. The paper explains some clarification on the relationship between several related known concepts in the differential geometry of TM, such as the system of general paths of Douglas, the nonlinear connections of Barthel, ano and Ishihara, as well as the nonhomogeneous connection of Grifone. The collection is suitable for mathematicians, geometricians, physicists, and academicians interested in differential geometry.

This text contains an elementary introduction to continuous groups and differential invariants; an extensive treatment of groups of motions in euclidean, affine, and riemannian geometry; more. Includes exercises and 62 figures.

This volume presents a collection of problems and solutions in differential geometry with applications. Both introductory and advanced topics are introduced in an easy-to-digest manner, with the materials of the volume being self-contained. In particular, curves, surfaces, Riemannian and pseudo-Riemannian manifolds, Hodge duality operator, vector fields and Lie series, differential forms, matrix-valued differential forms, Maurer–Cartan form, and the Lie derivative are covered. Readers will find useful applications to special and general relativity, Yang–Mills theory, hydrodynamics and field theory. Besides the solved problems, each chapter contains stimulating supplementary problems and software implementations are also included. The volume will not only benefit students in mathematics, applied mathematics and theoretical physics, but also researchers in the field of differential geometry. Request Inspection Copy

In this book we first review the ideas of Lie groupoid and Lie algebroid, and the associated concepts of connection. We next consider Lie groupoids of fibre morphisms of a fibre bundle, and the connections on such groupoids together with their symmetries. We also see how the infinitesimal approach, using Lie algebroids rather than Lie groupoids, and in particular using Lie algebroids of vector fields along the projection of the fibre bundle, may be of benefit. We then introduce Cartan geometries, together with a number of tools we shall use to study them. We take, as particular examples, the four classical types of geometry: affine, projective, Riemannian and conformal geometry. We also see how our approach can start to fit into a more general theory. Finally, we specialize to the geometries (affine and projective) associated with path spaces and geodesics, and consider their symmetries and other properties.

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This introductory text defines geometric structure by specifying parallel transport in an appropriate fiber bundle and focusing on simplest cases of linear parallel transport in a vector bundle. 1981 edition.

This book provides an introduction to the basic concepts in differential topology, differential geometry, and differential equations, and some of the main basic theorems in all three areas. This new edition includes new chapters, sections, examples, and exercises. From the reviews: "There are many books on the fundamentals of differential geometry, but this one is quite exceptional; this is not surprising for those who know Serge Lang's books." --EMS NEWSLETTER

A great book ... a necessary item in any mathematical library. --S. S. Chern, University of California A brilliant book: rigorous, tightly organized, and covering a vast amount of good mathematics. --Barrett O'Neill, University of California This is obviously a very valuable and well thought-out book on an important subject. --Andre Weil, Institute for Advanced Study The study of homogeneous spaces provides excellent insights into both differential geometry and Lie groups. In geometry, for instance, general theorems and properties will also hold for homogeneous spaces, and will usually be easier to understand and to prove in this setting. For Lie groups, a significant amount of analysis either begins with or reduces to analysis on homogeneous spaces, frequently on symmetric spaces. For many years and for many mathematicians, Sigurdur Helgason's classic Differential Geometry, Lie Groups, and Symmetric Spaces has been--and continues to be--the standard source for this material. Helgason begins with a concise, self-contained introduction to differential geometry. Next is a careful treatment of the foundations of the theory of Lie groups, presented in a manner that since 1962 has served as a model to a number of subsequent authors. This sets the stage for the introduction and study of symmetric spaces, which form the central part of the book. The text concludes with the classification of symmetric spaces by means of the Killing-Cartan classification of simple Lie algebras over $\mathbb{C}$ and Cartan's classification of simple Lie algebras over $\mathbb{R}$, following a method of Victor Kac. The excellent exposition is supplemented by extensive collections of useful exercises at the end of each chapter. All of the problems have either solutions or substantial hints, found at the back of the book. For this edition, the author has made corrections and added helpful notes and useful references. Sigurdur Helgason was awarded the Steele Prize for Differential Geometry, Lie Groups, and Symmetric Spaces and Groups and Geometric Analysis.

Writing this book, I had in my mind areader trying to get some knowledge of a part of the modern differential geometry. I concentrate myself on the study of sur faces in the Euclidean 3-space, this being the most natural object for investigation. The global differential geometry of surfaces in E3 is based on two classical results: (i) the ovaloids (i.e., closed surfaces with positive Gauss curvature) with constant Gauss or mean curvature are the spheres, (ü) two isometrie ovaloids are congruent. The results presented here show vast generalizations of these facts. Up to now, there is only one book covering this area of research: the Lecture Notes [3] written in the tensor slang. In my book, I am using the machinary of E. Cartan's calculus. It should be equivalent to the tensor calculus; nevertheless, using it I get better results (but, honestly, sometimes it is too complicated). It may be said that almost all results are new and belong to myself (the exceptions being the introductory three chapters, the few classical results and results of my post graduate student Mr. M. ÄFWAT who proved Theorems V.3.1, V.3.3 and VIII.2.1-6).

Elie Cartan's book Geometry of Riemannian Manifolds (1928) was one of the best introductions to his methods. It was based on lectures given by the author at the Sorbonne in the academic year 1925-26. A modernized and extensively augmented edition appeared in 1946 (2nd printing, 1951, and 3rd printing, 1988). Cartan's lectures in 1926-27 were different -- he introduced exterior forms at the very beginning and used extensively orthonormal frames throughout to investigate the geometry of Riemannian manifolds. In this course he solved a series of problems in Euclidean and non-Euclidean spaces, as well as a series of variational problems on geodesics. The lectures were translated into Russian in the book Riemannian Geometry in an Orthogonal Frame (1960). This book has many innovations, such as the notion of intrinsic normal differentiation and the Gaussian torsion of a submanifold in a Euclidean multidimensional space or in a space of constant curvature, an affine connection defined in a normal fiber bundle of a submanifold, etc. The only book of Elie Cartan that was not available in English, it has now been translated into English by Vladislav V Goldberg, the editor of the Russian edition.

Bundles, connections, metrics and curvature are the lingua franca of modern differential geometry and theoretical physics. Supplying graduate students in mathematics or theoretical physics with the fundamentals of these objects, this book would suit a one-semester course on the subject of bundles and the associated geometry.

In this book, the general theory of submanifolds in a multidimensional projective space is constructed. The topics dealt with include osculating spaces and fundamental forms of different orders, asymptotic and conjugate lines, submanifolds on the Grassmannians, different aspects of the normalization problems for submanifolds (with special emphasis given to a connection in the normal bundle) and the problem of algebraizability for different kinds of submanifolds, the geometry of hypersurfaces and hyperbands, etc. A series of special types of submanifolds with special projective structures are studied: submanifolds carrying a net of conjugate lines (in particular, conjugate systems), tangentially degenerate submanifolds, submanifolds with asymptotic and conjugate distributions etc. The method of moving frames and the apparatus of exterior differential forms are systematically used in the book and the results presented can be applied to the problems dealing with the linear subspaces or their generalizations. Graduate students majoring in differential geometry will find this monograph of great interest, as will researchers in differential and algebraic geometry, complex analysis and theory of several complex variables.

This book describes the life and achievements of the great French mathematician, Elie Cartan. Here readers will find detailed descriptions of Cartan's discoveries in Lie groups and algebras, associative algebras, differential equations, and differential geometry, as well of later developments stemming from his ideas. There is also a biographical sketch of Cartan's life. A monumental tribute to a towering figure in the history of mathematics, this book will appeal to mathematicians and historians alike.

Differential geometry began as the study of curves and surfaces using the methods of calculus. In time, the notions of curve and surface were generalized along with associated notions such as length, volume, and curvature. At the same time the topic has become closely allied with developments in topology. The basic object is a smooth manifold, to which some extra structure has been attached, such as a Riemannian metric, a symplectic form, a distinguished group of symmetries, or a connection on the tangent bundle. This book is a graduate-level introduction to the tools and structures of modern differential geometry. Included are the topics usually found in a course on differentiable manifolds, such as vector bundles, tensors, differential forms, de Rham cohomology, the Frobenius theorem and basic Lie group theory. The book also contains material on the general theory of connections on vector bundles and an in-depth chapter on semi-Riemannian geometry that covers basic material about Riemannian manifolds and Lorentz manifolds. An unusual feature of the book is the inclusion of an early chapter on the differential geometry of hyper-surfaces in Euclidean space. There is also a section that derives the exterior calculus version of Maxwell's equations. The first chapters of the book are suitable for a one-semester course on manifolds. There is more than enough material for a year-long course on manifolds and geometry.